ON-LINE ASSESSMENT IN UNDERGRADUATE MATHEMATICS

An experiment using the system CAL for generating multiple choice questions

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ABSTRACT

In this paper we report on two experiments of assessment of Linear Algebra courses, each involving more than 300 students, led at *Instituto Superior Técnico, Lisbon*, in 2000/2001 and 2001/2002. We used a system for authomatic grading and generation of multiple choice questions. In this way we were able to assess the students weekly.

Multiple choice questions were chosen because they are easy to grade automatically. However, this does not necessarily mean that the questions themselves are boring and uninteresting. The students have, of course, to provide the right answer, but they have enough time to find it. Each individual exercise list has a week time to be solved and students usually discuss the questions with their colleagues and teachers.

The most important goal of this on-line assessment model is not to assess the students but to provide them a weekly stimulus for learning the subjects teached at lectures at that time. The final grade takes these grades into account but also the grade of a final written exam and written tests.

The results were very convincing [3]. Students were highly motivated and kept asking specific questions about the subject matter. The rates of success in the course grading were higher than those of previous years.

We have used a system (CAL-Computer Aided Learning) that randomly generates different instances of the same template question. Six to eight template questions are organized in one exercise list. Each student is assigned a different instance of this exercise list.

Although CAL has proved to be very usefull it is not the best solution since it requires a considerable effort and knowledge of programming from the teacher who is implementing new questions. A simpler approach is to re-use and to slightly adapt template questions already available.

1 Introduction

Exercising in Mathematics, especially in basic courses such as Linear Algebra and Calculus, is important. The more a student is exposed to solving problems, the deeper is his/her understanding of a given subject. Moreover, the exercises a student tries to solve shape the way he/she understands the subject.

The web provides a way of displaying material that is more appealing to new generations of students, plus giving them the freedom to choose the time, place, and style of study. As instructors of Mathematics in an engineering school we are sensitive to all these possibilities offered by the web [1,2,4].

For that purpose it is necessary to have a system that is able to produce automatically many questions, to make these questions available in the form of web-based exercise lists and to automatically grade the answers.

In the next section 2 we present the system CAL that we used for the purposes just described, including a simple example that illustrates the main concepts. In section 3 we report on our experience using the system and in section 4 we conclude and refer to future directions of work.

2 Computer Aided Learning - CAL

The CAL system allows one to write template multiple choice questions, and generates random instances of these questions, thus producing individual web-based exercise lists for students to solve and that can be used for assessment, and as a training basis for first-year undergraduates.

During the last two years, we created a database of multiple choice questions on Linear Algebra. The text of each template question depends on parameters and is written in *Mathematica*, directly using the algebraic operations made available by *Mathematica*. The program randomly determines the parameters, and also determines the right answer. In this way we are able to get different instances of the same template questions. In the following we illustrate how to write template questions. We chose a very simple example that refers the basic concepts involved in a template question. We hope to convice the reader that this is not a too difficult task.

Another example is presented in order to show that it is possible to construct interesting questions (even in a multiple choice setting). This and other examples (in portuguese) are available in http://www.math.ist.utl.pt/∼cal2000.

2.1 Determinant of a Matrix

This first example is a very simple one. The student is asked to compute the determinant of a given matrix. The question has the following form:

Consider the matrix ■. Its determinant is:

1. *wrong answer 1*
2. *right answer*
3. *wrong answer 2*
4. *wrong answer 3*

The ■ represents a concrete matrix that is randomly chosen each time that a new instance of the question is needed. The possibility of choosing randomly a particular matrix (or other mathematical object) with certain properties is the main facility of the system that we use.

2.2 Types

In general one defines previously the *type* of the parameters to be randomly determined. In this example we firstly define the type of the entries (they can be any integer between -2 and 2) and the type of the matrix (a 3×3 matrix with those entries):

TypeOfEntries = INTERVAL[Integer, {-2, 2}];
TypeOfMatrix = MATRICES[TypeOfEntries][{i, 3}, {j, 3}];

In order to make life easier for the students (and to illustrate another construction) we refine the type of matrices and consider the subset of those matrices that satisfy the further condition that their determinant (in absolute value) is less or equal than 10:

NewTypeOfMatrix =
SUBSET[TypeOfMatrix, Function[m, Abs[Det[m]] <= 10], 1000];

(The number 1000 is the number of times the system will try to find a matrix satisfying these conditions).

These are all types we need for this example. A concrete instance of such a type is obtained by the command **RandomChoice** as shown in the next section.

2.3 Random instances and errors

Recall that we want the student to find the determinant of a certain matrix and then select the right answer from among four different ones. That means that we have to a) generate a matrix and determin its determinant and b) generate 3 other numbers to be used as "wrong determinants" of that matrix. The sequence

Matrix = RandomChoice[NewTypeOfMatrix];
determinant=Det[Matrix]

assigns a new matrix to the corresponding variable and then determins its determinant.

There are several ways to define the "wrong determinants". For simplicity these are the determinants of 3 new matrices. Since these numbers have to be all different and different from the right determinant we repeat the choice of new matrices until this condition is met.

DetError1 = DetError2 = DetError3 = determinant;
While[Not[determinant ≠ DetError1 ≠ DetError2 ≠ DetError3],
DetError1 = Det[RandomChoice[NewTypeOfMatrix]];
DetError2 =Det[RandomChoice[NewTypeOfMatrix]];
DetError3 = Det[RandomChoice[NewTypeOfMatrix]]]

At this point we have all data necessary for the question.

2.4 Text of the question

The text of the question (written in a text *Mathematica* cell) is simply:

Consider the matrix Matrix. *Its determinant is:*

Above *Matrix* is a *Mathematica Inline expression* meaning that the value of *Matrix* (the randomly determined matrix) will be placed at that position in the final text.

The right and wrong answers are treated in a similar way.

2.5 Generation of a question

By running a particular command (**SaveAsHTML**) on a *Mathematica* notebook containing the code previously described a new instance of the question is generated. We show two examples in the following:

Consider the matrix $\begin{pmatrix} 0 & -2 & 1 \\ 1 & -1 & 1 \\ 0 & 1 & -2 \end{pmatrix}$.

Its determinant is:

−3 0 2 1

Consider the matrix $\begin{pmatrix} 2 & 1 & 0 \\ -2 & 1 & 1 \\ -2 & -1 & 1 \end{pmatrix}$.

Its determinant is:

4 6 0 −1

Each example is shown with the list of choices (authomatically generated). The first is the right one. The list of choices is processed (see below) so that their order is randomly changed.

Besides the implementation of several "types of mathematical objects" specific packages for Calculus and Linear Algebra have been also developed by the authors.

2.6 Another example

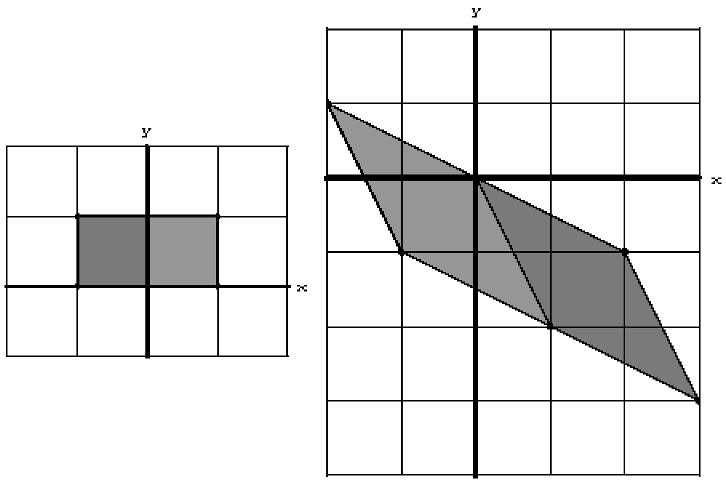
The following graphically appealing question is by no means easy to solve and shows that it is possible to have interesting questions also in a multiple choice setting.

The text reads:

Let $T : \mathbb{R}^2 \Rightarrow \mathbb{R}^2$ be a linear transformation that maps the paralelogram on the left to the one on the right. Find the cosine of the least angle between the eigenvectors of T.

1.

Considere a aplicação linear T de $\mathbb{R}^2$ em $\mathbb{R}^2$ que transforma o paralelogramo da figura à esquerda no da figura à direita. Qual o coseno do menor ângulo entre as direcções próprias da aplicação T ?



(1 valores)

- ○ 0
- ○ 1
- ○ $\frac{1}{\sqrt{10}}$
- ○ $\frac{2}{\sqrt{5}}$

2.7 Making the questions available

After the generation of enough instances of the exercises these are again randomly assigned to the students (identified by a username). The Perl script that does the random assignement is a modification of a preliminary version of *Web Assign* from Larry Martin and is also responsible to change the order of the choices of the multiple choice.

2.8 User Interface

The time given for the resolution of each individual list of exercises is decided by the teachers. We chose one week for each list. The student enters the system with his/her username and password and has access to his/hers individual exercise list. The selection of a choice is by clicking on the corresponding box (we chose always to have four possibilities). During the week the student can obtain a hard copy of the list of exercises, exit and return to the electronic list even without answering some or all questions. However, by clicking Submit, his/her present selected choices are sent to the system and considered final. An immediate feedback is given about how many right/wrong answers the student has submitted. Afterwords, the student can still return to the list, and select choices for other questions but not for those already considered final. At the end of the week the list of exercises is closed. The student can still access his/hers individual list of exercises and be informed on the right choices for the questions of that individual list of exercises. This quick feed-back helps identifying subjects that need further study and that he/she probably would not be aware otherwise.

3 Assessing First-year Undergraduates

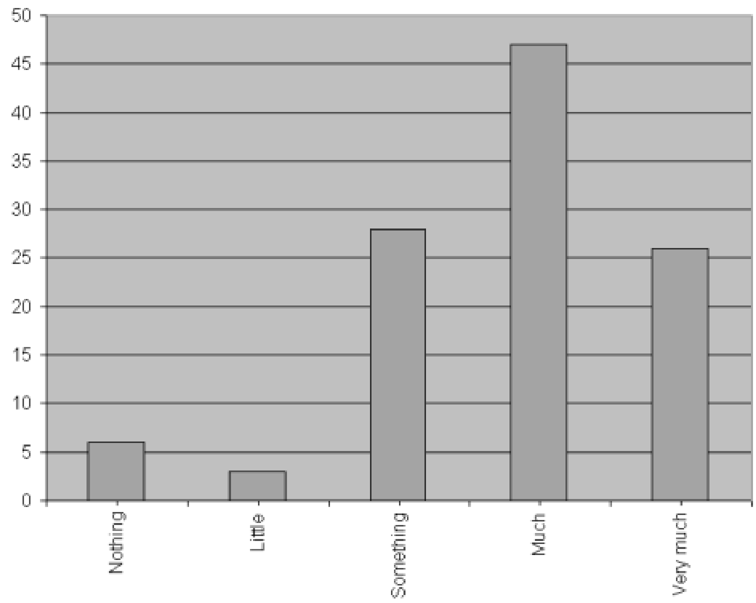
We used CAL in a large scale (400 students) for the first time in the Linear Algebra course of the first semester of 2000/2001.

The students answered eight lists of exercises (each open for a week) covering many different subjects on Linear Algebra. These exercises contributed 30% to the final grade in Linear Algebra. The other 70% of the assessment was traditional, that is, through grading of written tests and exams. We took great care that the questions in these exams and tests were at least of the same difficulty level of those of previous years.

The results were very convincing. Students were highly motivated and kept asking specific questions about the subject matter. The rates of success in the course grading were higher than those of previous years.

During an evaluation survey, students were highly positive about this experiment and its contribution to their own learning process. From about 100 volunteers, who answered the survey, 70% considered that the on-line exercises were very important to their understanding of the course content (see next table).

How much did the on-line exercises help you understanding the subjects?



This year the same model was used for the second time. Although we do not have final grades (since the grading process is not yet concluded), the results after the first written exam suggest an even better success rate.

4 Conclusions and Future Work

We have described an on-line assessment model whose main purpose is not to assess the students but to provide them a weekly stimulus for learning the subjects teached at lectures at that time. It improves learning by facing students with several exercises that they have to solve throughout the semester.

This helps mostly the less motivated students that, in other situations, loose track of the sujects after the first two weeks and only try to learn some days before the final exam (and fail). In principle this type of assessment might be done traditionally, without computers.

For the teacher the advantage of this system is the possibility of generating many different questions (a different exercise list for each student) and, furthermore, without the otherwise extreme effort to grade them. Moreover, the fact that the exercises are available electronically seems to be more apealling to students than traditional lists of exercises.

Our conviction, supported by the positive reaction and the rate of success of students, is that this or similar systems do help students to learn.

There are several directions of future work: a) to improve the system with more questions possibly about different subjects; b) to use the system not only for assessment but also for self-learning and c) to improve technically the system.

We are cooperating with the University of Madeira in order to reproduce and improve the assessment experience of Linear Algebra there, adapting and creating more template questions. This will improve the variety of the database of questions in this subject. In cooperation with the same university we plan to make available exams of mathematics of interest to secondary school students and teachers. Moreover, we are already constructing a database of questions (not only multiple choice ones) on Calculus II for first-year undergraduates. Cooperation with teams from other universities is welcome (our questions are in Portuguese but should be easy to translate).

On another direction we are planning to create an electronic textbook for secondary school mathematics that uses some template questions for ilustration and self-assessment purposes. In this case the questions are not only graded but a short explanation of the resolution should be given. The textbook is intended to help to fill the gap between secondary school mathematics and undergraduate courses.

We are currently improving the interface with the teacher in many ways: selecting the questions for a test in an easier way, entering student data and entering student scores in the grade book.

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